

Effect of heat transfer on the performance of thermoelectric generator-driven thermoelectric refrigerator system

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ABSTRACT

A model of thermoelectric generator-driven thermoelectric refrigerator with external heat transfer is proposed. The performance of the combined thermoelectric refrigerator device obeying Newton's heat transfer law is analyzed using the combination of finite time thermodynamics and non-equilibrium thermodynamics. Two analytical formulae for cooling load vs. working electrical current, and the coefficient of performance (COP) vs. working electrical current, are derived. For a fixed total heat transfer surface area of four heat exchangers, the allocations of the heat transfer surface area among the four heat exchangers are optimized for maximizing the cooling load and the coefficient of performance (COP) of the combined thermoelectric refrigerator device. For a fixed total number of thermoelectric elements, the ratio of number of thermoelectric elements of the generator to the total number of thermoelectric elements is also optimized for maximizing both the cooling load and the COP of the combined thermoelectric refrigerator device. The influences of thermoelectric element allocation and heat transfer area allocation are analyzed by detailed numerical examples. Optimum working electrical current for maximum cooling load and COP at different total number of thermoelectric elements and different total heat transfer area are obtained, respectively.

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1. Introduction

Semiconductor thermoelectric power generation, based on the Seebeck effect, and semiconductor thermoelectric cooling, based on the Peltier effect, have very interesting capabilities with respect to conventional power generation and cooling systems [1–3]. The absence of moving components results in an increase of reliability, a reduction of maintenance, and an increase of system life; the modularity allows for application in a wide-scale range without significant losses in performance; the absence of a working fluid avoids environmental dangerous leakage; and the noise reduction appears also to be an important feature. Thermoelectric generator and refrigerator have been used in military, aerospace, instrument, and industrial or commercial products, as a power generation or cooling devices for specific purposes. Many researchers are concerned about the physical properties of thermoelectric material and the manufacturing technique of thermoelectric modules. In addition to the improvement of the thermoelectric material and module, the system analysis and optimization of thermoelectric generator and refrigerator are equally important in designing high-performance thermoelectric generators and refrigerators.

In general, the conventional non-equilibrium thermodynamics [1,4] is used to analyze the performance of single-stage one- or multiple-element thermoelectric generators [5–13] and refrigerators [14–25]. Non-equilibrium thermodynamics is a progress of classical thermodynamics, it considers some specific phenomena such as Seebeck effect and Peltier effect. All objects of these researches are independent thermoelectric devices, that is, they generate direct-current power source for users (thermoelectric generator) or need a direct-current power source to provide direct current (thermoelectric refrigerator). In some special fields, the heat rejected from the thermal machine may drive a thermoelectric refrigerator through the use of a thermoelectric generator, so that the thermoelectric refrigerator does not need an independent power source. Such a new system is different from the traditional thermoelectric systems merely consisting of a thermoelectric generator and a refrigerator. These systems dispense with complicated pipelines and heat insulation, so they can be used in many special fields such as aircraft and submarine. Chen et al. [26] and Khattab and El Shenawy [27] built a model of this kind of combined system, i.e. single-stage thermoelectric refrigerator driven by single-stage thermoelectric generator, and analyzed the performance of the device.

The theory of finite time thermodynamics or entropy generation minimization [28–35] is a powerful tool for performance analysis and optimization of practice thermodynamic processes and

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Nomenclature

A	coefficient of system stable working electrical current equation	<i>Greek symbols</i>	
F	heat transfer surface areas of heat exchanger	α	Seebeck coefficients of P- and N- type semiconductor legs
f	heat transfer surface area ratio	ε	coefficient of performance (COP) of combined thermoelectric refrigerator device
I	working electrical current (A)	<i>Subscripts</i>	
K	thermal conductance of a semiconductor couple (W/K)	1	parameter of thermoelectric generator
k	heat transfer coefficient of heat exchanger (W/K)	2	parameter of thermoelectric refrigerator
M	total number of thermoelectric elements pairs of whole device	$H1$	heat source of thermoelectric generator
m	number of thermoelectric elements pairs of thermoelectric generator	$H2$	heat sink of thermoelectric refrigerator
n	number of thermoelectric elements pairs of thermoelectric refrigerator	$L1$	heat sink of thermoelectric generator
T	temperature (K)	$L2$	heat source of thermoelectric refrigerator
Q	rate at which heat is transferred (W)	s	practical solution of working electrical current equation
R	total internal electrical resistance of a semiconductor couple (Ω)	opt	optimum parameter
x	ratio of number of thermoelectric element pairs of the thermoelectric generator to total number of thermoelectric element pairs of the combined irreversible device		

devices. Finite time thermodynamics or entropy generation minimization is the method of modeling and optimization of various thermodynamic processes and devices that owe their thermodynamic imperfection to heat transfer, mass transfer, and fluid flow and other transport processes. It bridges the gaps not only between thermodynamic and heat transfer, mass transfer, fluid mechanics, and other transport science, but also between the physics and the engineering. It thermodynamically optimize performance of real finite-time and/or finite-size thermodynamic systems with the irreversibilities of heat transfer, fluid flow and mass transfer toward decreasing the irreversibility of the total system. Some authors have investigated the performance of thermoelectric generators [36–48] and thermoelectric refrigerator [49–57] using the combination of finite time thermodynamics and non-equilibrium thermodynamics. They analyzed the effect of finite-rate heat transfer between the thermoelectric device and its external heat reservoirs on the performance of single-element single-stage thermoelectric generators [36–43] and refrigerator [49–52]. They also investigated the characteristics of single-stage multi-element thermoelectric generators [44–48] and thermoelectric refrigerator [53–57] with the irreversibility of finite rate heat transfer, Joulean heat inside the thermoelectric device, and the heat leak through the thermoelectric couple leg. However, all of those were performed only for independent thermoelectric devices. There has been no investigation concerning the performance analysis and optimization for single-stage thermoelectric refrigerator driven by single-stage thermoelectric generator published in the open literature.

On the basis of the exo-reversible model of a single-stage thermoelectric refrigerator driven by a single-stage thermoelectric generator without external irreversibility built in Refs. [26,27], a model of thermoelectric generator-driven thermoelectric refrigerator with external heat transfer is built. The performance of the combined thermoelectric refrigerator device obeying Newton's heat transfer law is analyzed using the combination of finite time thermodynamics and non-equilibrium thermodynamics. Two analytical formulae for cooling load vs. working electrical current, and the coefficient of performance (COP) vs. working electrical current, are derived. For a fixed total heat transfer surface area of four heat exchangers, the allocations of the heat transfer surface area among the four heat exchangers are optimized for maximizing the cooling load and the coefficient of performance (COP) of the combined thermoelectric refrigerator device. For a fixed total number of

thermoelectric elements, the ratio of number of thermoelectric elements of the generator to the total number of thermoelectric elements is also optimized for maximizing both the cooling load and the COP of the combined thermoelectric refrigerator device. The influences of thermoelectric element allocation and heat transfer area allocation are analyzed by detailed numerical examples. Optimum working electrical current for maximum cooling load and COP at different total number of thermoelectric elements and different total heat transfer area are given, respectively.

2. Model of a thermoelectric generator-driven thermoelectric refrigerator system

A schematic diagram of a thermoelectric generator-driven thermoelectric refrigerator device is shown in Fig. 1. The device consists of an irreversible single-stage multi-element thermoelectric generator and an irreversible single-stage multi-element thermoelectric refrigerator in series with internal and external irreversibilities. The direct-current power source of the refrigerator is the direct-current power output of the generator.

The irreversible thermoelectric generator is composed of m pairs of thermoelectric elements. Each element is composed of P-type and N-type semiconductor legs. The thermoelectric power generation element is assumed to be insulated, both electrically and thermally, from its surroundings, except at the junction-reservoir contacts. The internal irreversibility is caused by Joulean electrical resistive loss and heat conduction loss through the semiconductor between the hot and cold junctions. The Joulean loss generates an internal heat I^2R , where R is the total internal electrical resistance of the semiconductor couple and I is the electrical current generating from the semiconductor couple. The conduction heat loss is $K(T_{H1} - T'_{L1})$, where K is the thermal conductance of the semiconductor couple, T_{H1} is the hot junction temperature, and T'_{L1} is the cold junction temperature. Finite rate heat transfers, i.e. the temperature differences $(T_{H1} - T'_{H1})$ and $(T'_{L1} - T_{L1})$, where T_{H1} and T_{L1} are the temperatures of the heat source and heat sink of the thermoelectric generator, respectively, cause the external irreversibility. For the thermoelectric generator, the rate of heat transfer at the hot junction is Q_{H1} , and the rate of heat transfer at the cold junction is Q_{L1} .

The irreversible thermoelectric refrigerator is composed of n pairs of thermoelectric elements. Each element is composed of

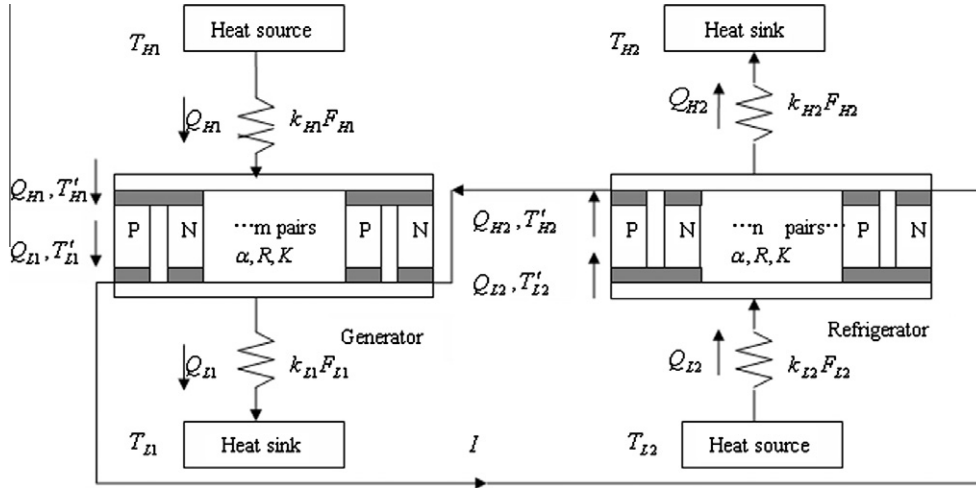


Fig. 1. Schematic diagram of the combined thermoelectric device.

P-type and N-type semiconductor legs. The thermoelectric refrigerating element is assumed to be insulated, both electrically and thermally, from its surroundings, except at the junction-reservoir contacts. The internal irreversibility is caused by Joulean electrical resistive loss and heat conduction loss through the semiconductor between the hot and cold junctions. The Joulean loss generates an internal heat $I^2 R$, where R is the total internal electrical resistance of the semiconductor couple and I is the electrical current generating from the semiconductor couple. The conduction heat loss is $K(T'_{H2} - T'_{L2})$, where K is the thermal conductance of the semiconductor couple, T'_{H2} is the hot junction temperature, and T'_{L2} is the cold junction temperature. Finite rate heat transfers, i.e. the temperature differences $(T_{H2} - T'_{H2})$ and $(T'_{L2} - T_{L2})$, where T_{H2} and T_{L2} are the temperatures of the heat sink and heat source of the thermoelectric refrigerator, respectively, cause the external irreversibility. For the thermoelectric refrigerator, the rate of heat transfer at the hot junction is Q_{H2} , and the rate of heat transfer at the cold junction is Q_{L2} .

Assume that the four heat exchangers among the hot and cold junctions of the thermoelectric refrigerator, thermoelectric generator and their respective reservoirs are counter-flow, and the heat conductance (product of heat transfer coefficient and heat transfer surface area) of the heat exchangers are $k_{H1}F_{H1}$, $k_{L1}F_{L1}$, $k_{H2}F_{H2}$ and $k_{L2}F_{L2}$, respectively, where k_{H1} , k_{L1} , k_{H2} and k_{L2} are the heat transfer

heat exchangers of the irreversible combined thermoelectric device is finite and $F = F_{H1} + F_{L1} + F_{H2} + F_{L2}$ holds.

Assuming that the heat transfers among the hot and cold junctions of the thermoelectric generator and the thermoelectric refrigerator and their respective reservoirs obey Newton's law gives:

$$Q_{H1} = k_{H1}F_{H1}(T_{H1} - T'_{H1}) = m \left[\alpha T_{H1} + K(T'_{H1} - T'_{L1}) - \frac{1}{2}I^2 R \right] \quad (1)$$

$$Q_{L1} = k_{L1}F_{L1}(T'_{L1} - T_{L1}) = m \left[\alpha T_{L1} + K(T'_{H1} - T'_{L1}) + \frac{1}{2}I^2 R \right] \quad (2)$$

$$Q_{H2} = k_{H2}F_{H2}(T'_{H2} - T_{H2}) = n \left[\alpha T_{H2} - K(T'_{H2} - T'_{L2}) + \frac{1}{2}I^2 R \right] \quad (3)$$

$$Q_{L2} = k_{L2}F_{L2}(T_{L2} - T'_{L2}) = n \left[\alpha T_{L2} - K(T'_{H2} - T'_{L2}) - \frac{1}{2}I^2 R \right] \quad (4)$$

where $\alpha = \alpha_p - \alpha_n$, α_p and α_n are the Seebeck coefficients of the P- and N- type semiconductor legs for each thermoelectric power generation and refrigeration element.

3. Performance analysis

Combining Eq. (1) with Eq. (2) gives the hot junction temperature T'_{H1} , and the cold junction temperature T'_{L1} of the thermoelectric generator:

$$T'_{H1} = \frac{0.5\alpha R m^2 I^3 - (0.5m + m^2 K) R I^2 + m\alpha T_{H1} I - mK(k_{H1}F_{H1}T_{H1} + k_{L1}F_{L1}T_{L1}) - k_{H1}F_{H1}k_{L1}F_{L1}T_{H1}}{m^2\alpha^2 I^2 + m\alpha(k_{H1}F_{H1} - k_{L1}F_{L1})I - mK(k_{H1}F_{H1} + k_{L1}F_{L1}) - k_{H1}F_{H1}k_{L1}F_{L1}} \quad (5)$$

$$T'_{L1} = \frac{-0.5\alpha R m^2 I^3 - (0.5m + m^2 K) R I^2 - m\alpha T_{H1} I - mK(k_{H1}F_{H1}T_{H1} + k_{L1}F_{L1}T_{L1}) - k_{H1}F_{H1}k_{L1}F_{L1}T_{H1}}{m^2\alpha^2 I^2 + m\alpha(k_{H1}F_{H1} - k_{L1}F_{L1})I - mK(k_{H1}F_{H1} + k_{L1}F_{L1}) - k_{H1}F_{H1}k_{L1}F_{L1}} \quad (6)$$

coefficients of the four heat exchangers, respectively, and F_{H1} , F_{L1} , F_{H2} and F_{L2} are the heat transfer surface areas of the four heat exchangers, respectively.

The total number (M) of thermoelectric element pairs of the irreversible combined thermoelectric device is finite and $M = m + n$ holds. The total heat transfer surface area (F) of the four

Combining Eq. (3) with Eq. (4) gives the hot junction temperature T'_{H2} and the cold junction temperature T'_{L2} of the thermoelectric refrigerator.

$$T'_{H2} = \frac{-0.5\alpha R n^2 I^3 - (0.5n + n^2 K) R I^2 - n\alpha T_{H2} I - nK(k_{H2}F_{H2}T_{H2} + k_{L2}F_{L2}T_{L2}) - k_{H2}F_{H2}k_{L2}F_{L2}T_{H2}}{n^2\alpha^2 I^2 + n\alpha(k_{L2}F_{L2} - k_{H2}F_{H2})I - nK(k_{H2}F_{H2} + k_{L2}F_{L2}) - k_{H2}F_{H2}k_{L2}F_{L2}} \quad (7)$$

$$T'_{L2} = \frac{-0.5\alpha Rn^2 I^3 - (0.5n + n^2 K)RI^2 + n\alpha T_{H2}I - nK(k_{H2}F_{H2}T_{H2} + k_{L2}F_{L2}T_{L2}) - k_{H2}F_{H2}k_{L2}F_{L2}T_{H2}}{n^2\alpha^2 I^2 + n\alpha(k_{L2}F_{L2} - k_{H2}F_{H2})I - nK(k_{H2}F_{H2} + k_{L2}F_{L2}) - k_{H2}F_{H2}k_{L2}F_{L2}} \quad (8)$$

Substituting Eqs. (5)–(8) into Eqs. (1)–(4) yields:

$$Q_{H1} = k_{H1}F_{H1}\{T_{H1} - [0.5\alpha Rm^2 I^3 - (0.5m + m^2 K)RI^2 + m\alpha T_{H1}I - mK(k_{H1}F_{H1}T_{H1} + k_{L1}F_{L1}T_{L1}) - k_{H1}F_{H1}k_{L1}F_{L1}T_{H1}]/[m^2\alpha^2 I^2 + m\alpha(k_{H1}F_{H1} - k_{L1}F_{L1})I - mK(k_{H1}F_{H1} + k_{L1}F_{L1}) - k_{H1}F_{H1}k_{L1}F_{L1}]\} \quad (9)$$

$$Q_{L1} = k_{L1}F_{L1}\{-0.5\alpha Rm^2 I^3 - (0.5m + m^2 K)RI^2 - m\alpha T_{H1}I - mK(k_{H1}F_{H1}T_{H1} + k_{L1}F_{L1}T_{L1}) - k_{H1}F_{H1}k_{L1}F_{L1}T_{H1}]/[m^2\alpha^2 I^2 + m\alpha(k_{H1}F_{H1} - k_{L1}F_{L1})I - mK(k_{H1}F_{H1} + k_{L1}F_{L1}) - k_{H1}F_{H1}k_{L1}F_{L1}] - T_{L1}\} \quad (10)$$

$$Q_{H2} = k_{H2}F_{H2}\{T_{H2} - [-0.5\alpha Rn^2 I^3 - (0.5n + n^2 K)RI^2 - n\alpha T_{H2}I - nK(k_{H2}F_{H2}T_{H2} + k_{L2}F_{L2}T_{L2}) - k_{H2}F_{H2}k_{L2}F_{L2}T_{H2}]/[n^2\alpha^2 I^2 + n\alpha(k_{L2}F_{L2} - k_{H2}F_{H2})I - nK(k_{H2}F_{H2} + k_{L2}F_{L2}) - k_{H2}F_{H2}k_{L2}F_{L2}]\} \quad (11)$$

$$Q_{L2} = k_{L2}F_{L2}\{-0.5\alpha Rn^2 I^3 - (0.5n + n^2 K)RI^2 + n\alpha T_{H2}I - nK(k_{H2}F_{H2}T_{H2} + k_{L2}F_{L2}T_{L2}) - k_{H2}F_{H2}k_{L2}F_{L2}T_{H2}]/[n^2\alpha^2 I^2 + n\alpha(k_{L2}F_{L2} - k_{H2}F_{H2})I - nK(k_{H2}F_{H2} + k_{L2}F_{L2}) - k_{H2}F_{H2}k_{L2}F_{L2}] - T_{L2}\} \quad (12)$$

The overall system is a closed loop circuit, and the heat flow of the system is in balance, one has:

$$Q_{H1} + Q_{L2} = Q_{L1} + Q_{H2} \quad (13)$$

Substituting Eqs. (9)–(12) into Eq. (13) and re-arranging the results yields the equation that the system stable working electrical current should be satisfied:

$$A_4 I^4 + A_3 I^3 + A_2 I^2 + A_1 I + A_0 = 0 \quad (14)$$

where

$$A_4 = \alpha^3 m^2 n^2 R(k_{L1}F_{L1} - k_{H1}F_{H1} - k_{L2}F_{L2} + k_{H2}F_{H2}) \quad (15)$$

$$A_3 = \alpha^2 mn[mn(2k_{H2}F_{H2}KR + 2k_{H2}F_{H2}T_{H2}\alpha^2 + 2k_{L2}F_{L2}KR + 2k_{H1}F_{H1}KR + 2k_{L2}F_{L2}T_{L2}\alpha^2 + 2\alpha^2 k_{L1}F_{L1}T_{L1} + 2k_{L1}F_{L1}KR + 2\alpha^2 k_{H1}F_{H1}T_{H1}) + m(k_{H2}F_{H2}Rk_{H1}F_{H1} - k_{H2}F_{H2}Rk_{L1}F_{L1} + k_{L2}F_{L2}Rk_{L1}F_{L1} - k_{L2}F_{L2}Rk_{H1}F_{H1} + 2k_{L2}F_{L2}k_{H2}F_{H2}R) + n(-k_{H2}F_{H2}Rk_{L1}F_{L1} + 2k_{H1}F_{H1}k_{L1}F_{L1}R - k_{L2}F_{L2}Rk_{H1}F_{H1} + k_{L2}F_{L2}Rk_{L1}F_{L1} + k_{H2}F_{H2}Rk_{H1}F_{H1})] \quad (16)$$

$$A_2 = \alpha[m^2 n(-3k_{L1}F_{L1}KRk_{H2}F_{H2} - 2\alpha^2 k_{L1}F_{L1}T_{L1}k_{H2}F_{H2} + 2\alpha^2 k_{L1}F_{L1}T_{L1}k_{L2}F_{L2} + 2\alpha^2 k_{H1}F_{H1}T_{H1}k_{L2}F_{L2} + k_{L1}F_{L1}KRk_{L2}F_{L2} - 2\alpha^2 k_{H1}F_{H1}T_{H1}k_{H2}F_{H2} - 2k_{L2}F_{L2}T_{L2}\alpha^2 k_{H2}F_{H2} + 2k_{H2}F_{H2}T_{H2}\alpha^2 k_{L2}F_{L2} - k_{H1}F_{H1}KRk_{H2}F_{H2} + 3k_{H1}F_{H1}KRk_{L2}F_{L2}) + mn^2(-k_{L1}F_{L1}KRk_{L2}F_{L2} + 2\alpha^2 k_{L2}F_{L2}T_{L2}k_{H1}F_{H1})$$

$$+ 3k_{H1}F_{H1}KRk_{L2}F_{L2} - 2\alpha^2 k_{H1}F_{H1}T_{H1}k_{L1}F_{L1} + 2\alpha^2 k_{H2}F_{H2}T_{H2}k_{H1}F_{H1} - k_{L2}F_{L2}T_{L2}\alpha^2 k_{L1}F_{L1} + k_{H1}F_{H1}KRk_{H2}F_{H2} - 3k_{L1}F_{L1}KRk_{H2}F_{H2} + 2\alpha^2 k_{L1}F_{L1}T_{L1}k_{H1}F_{H1} - 2k_{H2}F_{H2}T_{H2}\alpha^2 k_{L1}F_{L1}) + m^2(-k_{L1}F_{L1}Rk_{H2}F_{H2}k_{L2}F_{L2} - k_{H1}F_{H1}Rk_{H2}F_{H2}k_{L2}F_{L2}) + n^2(k_{L2}F_{L2}Rk_{L1}F_{L1}k_{H1}F_{H1} - k_{H2}F_{H2}Rk_{L1}F_{L1}k_{H1}F_{H1}) + mn(2k_{L2}F_{L2}Rk_{L1}F_{L1}k_{H1}F_{H1} + 2k_{L1}F_{L1}Rk_{H2}F_{H2}k_{L2}F_{L2} + 2k_{H2}F_{H2}Rk_{L1}F_{L1}k_{H1}F_{H1} + 2k_{H1}F_{H1}Rk_{H2}F_{H2}k_{L2}F_{L2})] \quad (17)$$

$$A_1 = -2m^2 nK(k_{L2}F_{L2} + k_{H2}F_{H2})(k_{L1}F_{L1}KR + Kk_{H1}F_{H1}R + \alpha^2 k_{L1}F_{L1}T_{L1} + \alpha^2 k_{H1}F_{H1}T_{H1}) - 2m n^2 K(k_{H1}F_{H1} + k_{L1}F_{L1})(KRk_{L2}F_{L2} + KRk_{H2}F_{H2} + \alpha^2 k_{H2}F_{H2}T_{H2} + \alpha^2 k_{L2}F_{L2}T_{L2}) - 2m^2 k_{L2}F_{L2}k_{H2}F_{H2}(KRk_{L1}F_{L1} + KRk_{H1}F_{H1} + \alpha^2 k_{L1}F_{L1}T_{L1} + \alpha^2 k_{H1}F_{H1}T_{H1}) - 2n^2 k_{L1}F_{L1}k_{H1}F_{H1}(KRk_{L2}F_{L2} + KRk_{H2}F_{H2} + \alpha^2 k_{H2}F_{H2}T_{H2} + \alpha^2 k_{L2}F_{L2}T_{L2}) + 2mn[\alpha^2(k_{H1}F_{H1}k_{L1}F_{L1}k_{H2}F_{H2}T_{H1} - k_{L2}F_{L2}k_{H2}F_{H2}k_{H1}F_{H1}T_{L2} + k_{L1}F_{L1}k_{H1}F_{H1}k_{L2}F_{L2}T_{L1} - k_{H2}F_{H2}k_{L2}F_{L2}k_{L1}F_{L1}T_{H2} - k_{L1}F_{L1}k_{H1}F_{H1}k_{H2}F_{H2}T_{L1} + k_{L2}F_{L2}k_{H2}F_{H2}k_{L1}F_{L1}T_{L2} - k_{H1}F_{H1}k_{L1}F_{L1}k_{L2}F_{L2}T_{H1} + k_{H2}F_{H2}k_{L2}F_{L2}k_{H1}F_{H1}T_{H2}) - RK(k_{H2}F_{H2}k_{L1}F_{L1}k_{H1}F_{H1} + k_{L2}F_{L2}k_{H2}F_{H2}k_{H1}F_{H1} + k_{L2}F_{L2}k_{L1}F_{L1}k_{H1}F_{H1} + k_{L1}F_{L1}k_{L2}F_{L2}k_{H2}F_{H2})] - 2mRk_{H1}F_{H1}k_{L1}F_{L1}k_{H2}F_{H2}k_{L2}F_{L2} - 2nRk_{L2}F_{L2}k_{H2}F_{H2}k_{L1}F_{L1}k_{H1}F_{H1} \quad (18)$$

$$A_0 = 2mn(Kk_{L1}F_{L1}k_{H1}F_{H1}T_{H1}k_{H2}F_{H2} - Kk_{L1}F_{L1}k_{H1}F_{H1}T_{L1}k_{H2}F_{H2} + Kk_{L1}F_{L1}k_{H1}F_{H1}T_{H1}k_{L2}F_{L2} - Kk_{L1}F_{L1}k_{H1}F_{H1}T_{L1}k_{L2}F_{L2} - Kk_{L2}F_{L2}k_{H2}F_{H2}T_{H2}k_{L1}F_{L1} + Kk_{L2}F_{L2}k_{H2}F_{H2}T_{L2}k_{H1}F_{H1} + Kk_{L2}F_{L2}T_{L2}k_{H2}F_{H2}k_{L1}F_{L1} - Kk_{L2}F_{L2}k_{H2}F_{H2}T_{H2}k_{H1}F_{H1}) + 2m(k_{H1}F_{H1}k_{L1}F_{L1}k_{H2}F_{H2}k_{L2}F_{L2}T_{H1} - k_{L1}F_{L1}k_{H1}F_{H1}k_{H2}F_{H2}k_{L2}F_{L2}T_{L1}) + 2n(k_{L2}F_{L2}T_{L2}k_{H2}F_{H2}k_{L1}F_{L1}k_{H1}F_{H1} - k_{H2}F_{H2}k_{L2}F_{L2}k_{L1}F_{L1}k_{H1}F_{H1}T_{H2}) \quad (19)$$

For the given parameters, one can obtain four theoretical solutions by solving Eq. (14). Analysis shown that there is only one solution I_s , which satisfies $I > 0$, $Q_{H1} > 0$, $Q_{L1} > 0$, $Q_{H2} > 0$, and $Q_{L2} > 0$. Upon that one has the cooling load and COP of the combined thermoelectric device as follows:

$$Q_{L2} = k_{L2}F_{L2}\left\{\left[-0.5\alpha Rn^2 I_s^3 - (0.5n + n^2 K)RI_s^2 + n\alpha T_{H2}I_s - nK(k_{H2}F_{H2}T_{H2} + k_{L2}F_{L2}T_{L2}) - k_{H2}F_{H2}k_{L2}F_{L2}T_{H2}\right]/\left[n^2\alpha^2 I_s^2 + n\alpha(k_{L2}F_{L2} - k_{H2}F_{H2})I_s - (k_{H2}F_{H2} + k_{L2}F_{L2}) - k_{H2}F_{H2}k_{L2}F_{L2}\right] - T_{L2}\right\} \quad (20)$$

$$\varepsilon = Q_{L2}/Q_{H1}$$

$$= \frac{k_{L2}F_{L2} \left\{ \left[-0.5\alpha R n^2 I_s^3 - (0.5n + n^2 K) R I_s^2 + n\alpha T_{H2} I_s - nK(k_{H2}F_{H2}T_{H2} + k_{L2}F_{L2}T_{L2}) - k_{H2}F_{H2}k_{L2}F_{L2}T_{H2} \right] / \left[n^2\alpha^2 I_s^2 + n\alpha(k_{L2}F_{L2} - k_{H2}F_{H2})I_s - nK(k_{H2}F_{H2} + k_{L2}F_{L2}) - k_{H2}F_{H2}k_{L2}F_{L2} \right] - T_{L2} \right\}}{k_{H1}F_{H1} \left\{ T_{H1} - \left[0.5\alpha R m^2 I_s^3 - (0.5m + m^2 K) R I_s^2 + m\alpha T_{H1} I_s - mK(k_{H1}F_{H1}T_{H1} + k_{L1}F_{L1}T_{L1}) - k_{H1}F_{H1}k_{L1}F_{L1}T_{H1} \right] / \left[m^2\alpha^2 I_s^2 + m\alpha(k_{H1}F_{H1} - k_{L1}F_{L1})I_s - mK(k_{H1}F_{H1} + k_{L1}F_{L1}) - k_{H1}F_{H1}k_{L1}F_{L1} \right] \right\}} \quad (21)$$

Obviously, besides the performance parameters of thermoelectric element pairs (α , R and K), parameters concerning the heat transfer (k_{H1} , k_{L1} , k_{H2} , k_{L2} , F_{H1} , F_{L1} , F_{H2} and F_{L2}) would have influences on the device performance.

If $k_{H1}F_{H1} = k_{L1}F_{L1} = k_{H2}F_{H2} = k_{L2}F_{L2} \rightarrow \infty$, $T_{H1} = T'_{H1}$, $T_{L1} = T'_{L1}$, $T_{H2} = T'_{H2}$ and $T_{L2} = T'_{L2}$, Eqs. (20) and (21) become the results of conventional non-equilibrium thermodynamic analysis [26,27].

4. Numerical examples

4.1. Design variables and fixed parameters

For independent thermoelectric generator and thermoelectric refrigerator devices, there is an optimum heat transfer area allocation between the high temperature side and the low temperature

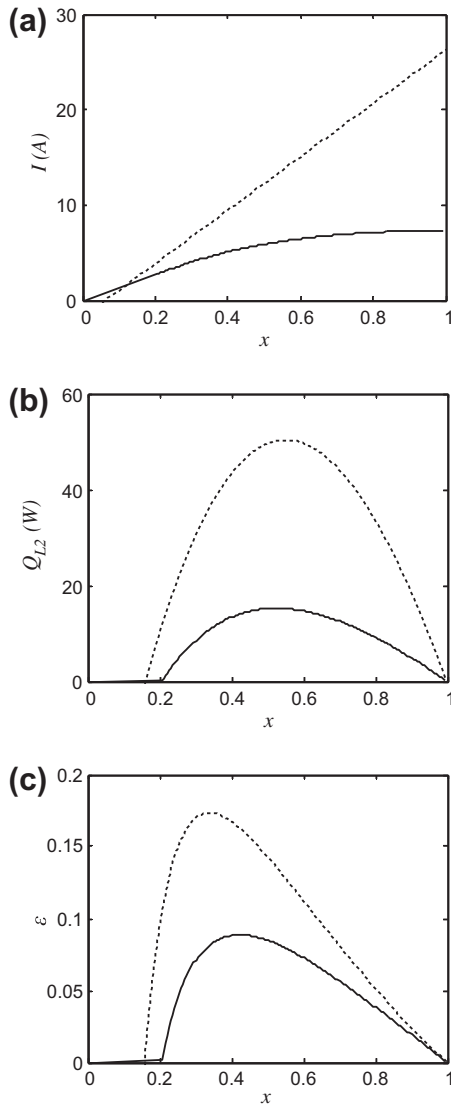


Fig. 2. Working electrical current (a), cooling load (b) and COP (c) vs. the ratio of number of thermoelectric elements. The solid line represents result obtained by using the combination of finite time thermodynamics and non-equilibrium thermodynamics. The dotted line represents result obtained by using non-equilibrium thermodynamics, i.e. effects of heat transfer are not taken into account.

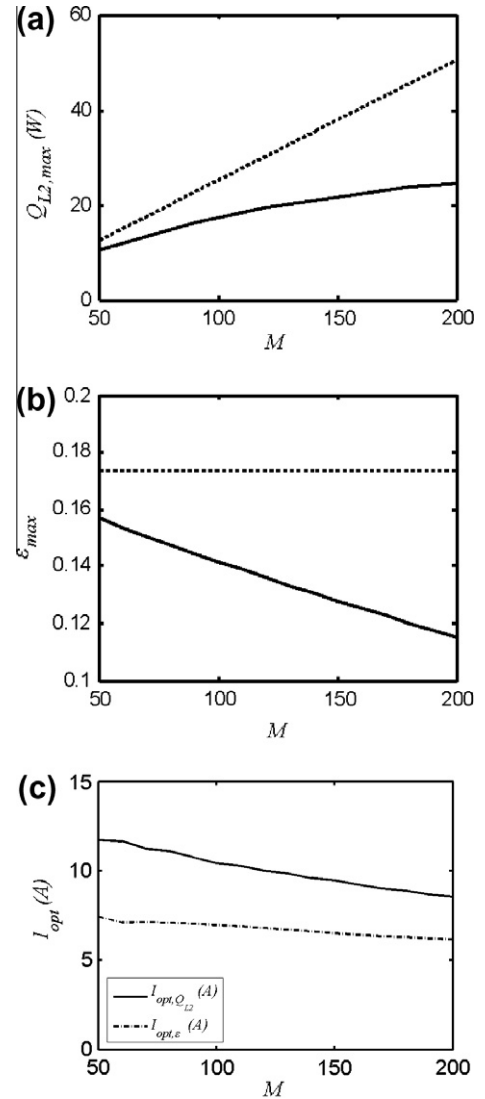


Fig. 3. Maximum cooling load (a), maximum COP (b) and optimum working electrical currents (c) vs. total number of thermoelectric elements. In (a) and (b), the solid line represents result obtained by using the combination of finite time thermodynamics and non-equilibrium thermodynamics while the dotted line represents result obtained by using non-equilibrium thermodynamics, i.e. effects of heat transfer are not taken into account. In (c), the solid line represents optimum working electrical current for maximum cooling load and the dash-dot line represents optimum working electrical current for maximum COP.

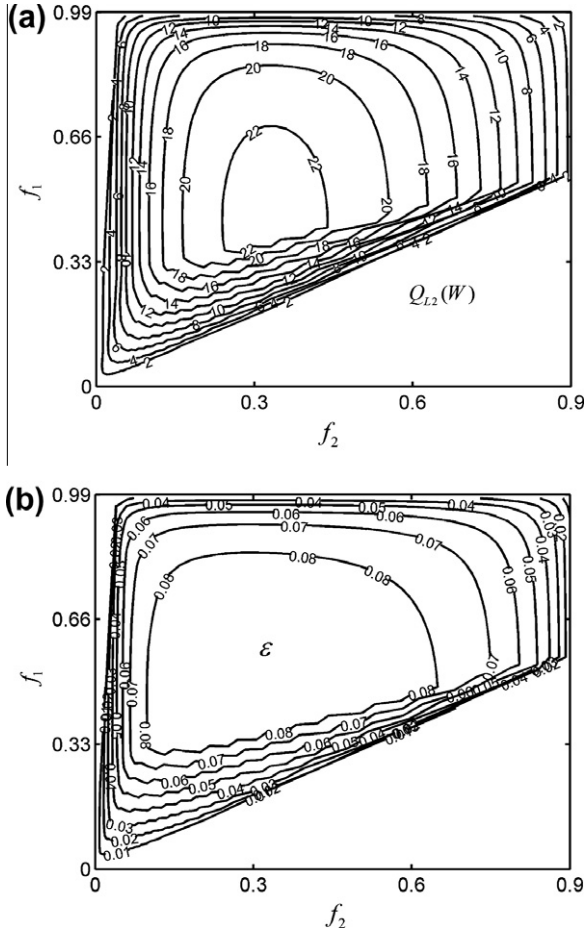


Fig. 4. Contour maps of the cooling load (a) and COP (b) vs. ratios of heat transfer surface area. f_1 is generator heat transfer surface area ratio, and f_2 is refrigerator heat transfer surface area ratio.

side heat exchangers for the fixed total heat transfer area of the thermoelectric generator or thermoelectric refrigerator. For the combined thermoelectric device, besides optimum heat transfer area allocation between the high temperature side and the low temperature side, there is an optimum heat transfer area allocation between the thermoelectric generator and the thermoelectric refrigerator for the fixed total heat transfer area of the whole device.

In order to describe the allocation of the heat transfer area, three ratios of heat transfer surface area are defined: total heat transfer surface area ratio $f = F_1/F$, where $F_1 = F_{H1} + F_{L1}$, i.e. the ratio of the heat transfer surface area of the thermoelectric generator to the total heat transfer surface area of the combined irreversible device; generator heat transfer surface area ratio $f_1 = F_{H1}/F_1$, i.e. the ratio of the heat transfer surface area of the high-temperature side heat exchangers of the thermoelectric generator to the total heat transfer surface area of the thermoelectric generator; and refrigerator heat transfer surface area ratio $f_2 = F_{H2}/F_2$, where $F_2 = F_{H2} + F_{L2}$, i.e. the ratio of the heat transfer surface area of the high-temperature side heat exchangers of the thermoelectric refrigerator to the total heat transfer surface area of the thermoelectric refrigerator. Then, one has $F_{H1} = ff_1F$, $F_{L1} = f(1 - f_1)F$, $F_{H2} = (1 - f)f_2F$ and $F_{L2} = (1 - f)(1 - f_2)F$.

In order to describe the allocation of the thermoelectric element pairs, one ratio of numbers of thermoelectric element pairs is defined: $x = m/M$, i.e. the ratio of number of thermoelectric element pairs of the thermoelectric generator to total number of thermoelectric element pairs of the combined irreversible device. Then, one has $m = xM$ and $n = (1 - x)M$.

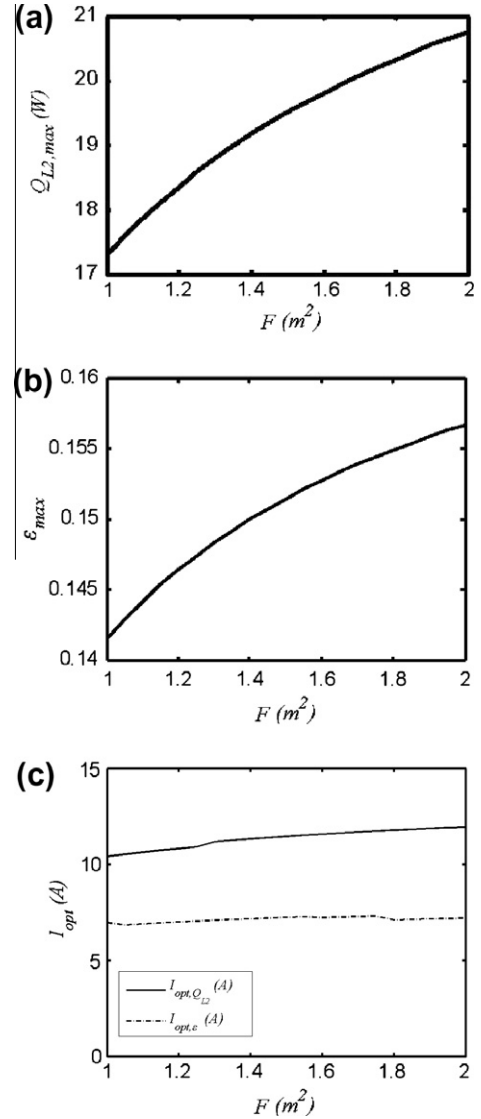


Fig. 5. Maximum cooling load (a), maximum COP (b) and optimum working electrical currents (c) vs. total heat transfer surface area. The solid line represents optimum working electrical current for maximum cooling load and the dash dot line represents optimum working electrical current for maximum COP.

Obviously, value ranges of the variables x, f, f_1, f_2 are $[0, 1]$.

Numerical calculations are performed in order to analyze and optimize the performance of the thermoelectric refrigerator driven by a thermoelectric generator. In the calculations, $T_{H1} = 450$ K, $T_{L1} = 300$ K, $T_{H2} = 300$ K, $T_{L2} = 290$ K, $k_{H1} = 60$ W/K, $k_{L1} = 15$ W/K, $k_{H2} = 240$ W/K, $k_{L2} = 120$ W/K, $\alpha = 2.1 \times 10^{-4}$ V/K, $K = 1.6 \times 10^{-2}$ W/K, and $R = 1.2 \times 10^{-3} \Omega$ are set [53].

4.2. Effects of total number and allocation of thermoelectric elements

The system working electrical current (I), cooling load (Q_{L2}) and COP (ε) vs. ratio of numbers of thermoelectric elements (x) are shown in Fig. 2 by solid lines, respectively. In the calculations, $M = 200, F = 1$ m², $f = 0.6$, $f_1 = 0.7$, and $f_2 = 0.4$ are set. There exist two optimum working electrical currents corresponding to maximum cooling load ($Q_{L2,max}$) and maximum COP (ε_{max}). The maximum cooling load and maximum COP vs. the total number of thermoelectric elements (M) are shown in Fig. 3a and b by solid lines, respectively. In order to compare the results obtained by using the combination of finite time thermodynamics and

non-equilibrium thermodynamics with these obtained by using non-equilibrium thermodynamics, the non-equilibrium thermodynamic results, i.e. the effects of heat transfer are not taken into account, are also shown in Figs. 2 and 3 by dotted lines. One can see that the effects of heat transfer are obvious and should be considered in the performance analysis and optimization of the combined irreversible thermoelectric devices.

The optimum working electrical current ($I_{opt,Q_{L2}}$) corresponding to maximum cooling load ($Q_{L2,max}$) and the optimum working electrical current ($I_{opt,\varepsilon}$) corresponding to maximum COP (ε_{max}) vs. the total number of thermoelectric element pairs (M) are shown in Fig. 3c.

One can see that the optimum working electrical current (I) is smaller and no longer proportional to x when effects of heat transfer are taken into account. I increases monotonically and the slope of the curve decreases monotonically with the increase of x . I changes little when x is near 1.

Practical cooling load (Q_{L2}) and COP (ε) is smaller. The minimum x , i.e. the ratio of number of thermoelectric elements corresponding the zero cooling load and zero COP is bigger when the effects of heat transfer are taken into account. However, heat transfer has little influence on optimum x , i.e. the ratios of number of thermoelectric elements corresponding the maximum cooling load and maximum COP.

The maximum cooling load ($Q_{L2,max}$) is smaller and no longer proportional to M when the effects of heat transfer are taken into account. $Q_{L2,max}$ increases monotonically and the slope of curve decreases monotonically with the increase of M . The maximum COP (ε_{max}) is no longer constant and decreases monotonically with the increase of M .

Both the optimum working currents $I_{opt,Q_{L2}}$ and $I_{opt,\varepsilon}$ decrease with the increase of total number of thermoelectric elements, and $I_{opt,Q_{L2}} \geq I_{opt,\varepsilon}$ holds.

4.3. Effects of total heat transfer area and allocation heat transfer area

Fig. 4 shows contour lines of the cooling load (Q_{L2}) and the COP (ε) vs. the heat transfer surface area ratios f_1 and f_2 , respectively, with $M = 100$, $F = 1 \text{ m}^2$, and $f = 0.6$.

One can see that there are an optimum f_1 and an optimum f_2 corresponding maximum cooling load $Q_{L2,max}$ or maximum COP ε_{max} , respectively. In fact, for fixed total number of thermoelectric elements M and total heat transfer area F , there are optimum parameters x , f , f_1 and f_2 corresponding the maximum cooling load $Q_{L2,max}$ or maximum COP ε_{max} , respectively.

Fig. 5 shows the maximum cooling load $Q_{L2,max}$, the maximum COP ε_{max} , the optimum working electrical current ($I_{opt,Q_{L2}}$) corresponding to maximum cooling load, and the optimum working electrical current ($I_{opt,\varepsilon}$) corresponding to maximum COP vs. the total heat transfer surface area F , respectively, with $M = 100$. One can see that both the maximum cooling load and COP increases monotonically with the increase of F . The optimum working currents $I_{opt,Q_{L2}}$ increases with the increase of total heat transfer area F while $I_{opt,\varepsilon}$ changes little and $I_{opt,Q_{L2}} \geq I_{opt,\varepsilon}$ holds.

Calculations show that when total heat transfer area changes, the optimum variables x , f , f_1 and f_2 remain constant approximately.

5. Conclusion

A model of internal and external irreversible thermoelectric generator-driven thermoelectric refrigerator is presented in this paper by using a combination of finite time thermodynamics and non-equilibrium thermodynamics. Two analytical formulae describing the cooling load vs. working electrical current, and the

coefficient of performance (COP) vs. working electrical current are derived. The performance optimization of the combined device is performed by searching for the optimum allocation of heat transfer surface area of the four heat exchangers and the optimum allocation of the number of thermoelectric element pairs based on the optimization of working electrical current. All the parameters should be considered in the design and application of practice thermoelectric devices in order to obtain the maximum economy benefit.

The results show that when effects of heat transfer are taken into account, the working electrical current is smaller than that by non-equilibrium thermodynamics. The cooling load is smaller and no longer proportional to total number of thermoelectric elements while the COP is no longer constant and decreases monotonically with the increase of total number of thermoelectric elements.

For the fixed total number of thermoelectric elements and total heat transfer surface area, there are optimum variables, i.e. ratio of numbers of thermoelectric element pairs x , total heat transfer surface area ratio f , generator heat transfer surface area ratio f_1 and refrigerator heat transfer surface area ratio f_2 , corresponding the maximum cooling load or maximum COP, respectively.

Heat exchangers are necessary in thermoelectric generator-driven thermoelectric refrigerator devices. There is always external heat transfer loss in the devices. Conventional non-equilibrium thermodynamic analysis cannot take into account the external heat transfer factor. The combination of finite time thermodynamics and non-equilibrium thermodynamic gives a comprehensive analysis and optimization of the devices by taking account of external and internal irreversibilities and obtains optimal performance characteristics closer to practice. The results obtained herein may provide guidelines for the design and application of practical combined thermoelectric devices. Some experiments will be made to confirm the performance of the combined device in the later work by the way of Ref. [53].

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